



## Elastic (indistinguishability) metrics for Location Privacy

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joint work with

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# Scope

## Privacy for LBS

Reducing accuracy

Goal: limited semantic inference  
(not anonymity)

## Utility

$f(\text{accuracy})$



# Privacy Definition

Mechanism

$$x \longrightarrow \mathcal{M} \longrightarrow z$$

# Privacy Definition

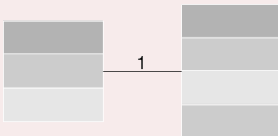
## Mechanism

$$x \longrightarrow \mathcal{M} \longrightarrow z$$

## DP: Differential Privacy

$$P[z \mid x] \leq e^{\epsilon} P[z \mid x'] \quad \forall x, x'. x \sim x'$$

[Dwork, McSherry, Nissim, Smith: Calibrating noise to sensitivity in private data analysis. TCC'06]



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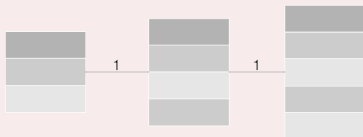
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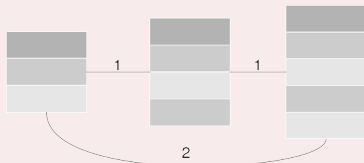
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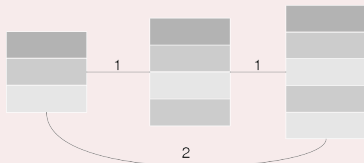
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## $d_{\mathcal{X}}$ -privacy

$$P[z \mid x] \leq e^{d_{\mathcal{X}}(x, x')} P[z \mid x'] \quad \forall x, x'$$

[Chatzikokolakis, Andres, Bordenabe, Palamidessi: Broadening the Scope of Differential Privacy Using Metrics. PETS'13]

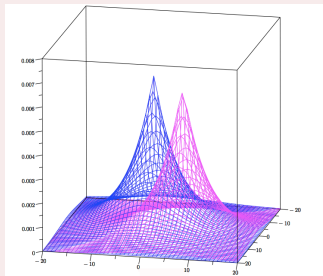


# Geo-indistinguishability

Metric: scaled Euclidean

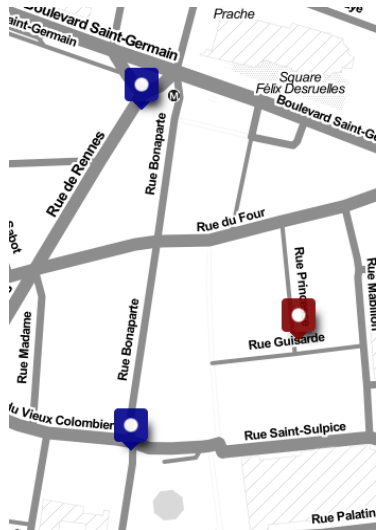
$$d_X(x, x') = \epsilon \cdot d_E(x, x')$$

Mechanism: Planar Laplacian



[Andrés, Bordenabe, Chatzikokolakis, Palamidessi: Geo-indistinguishability:

differential privacy for location-based systems. CCS'13]



# Family picture

$d_x$ -privacy

Differential Privacy

geo-indistinguishability

# Family picture

$d_x$ -privacy

Differential Privacy

geo-indistinguishability  
OptQL  
Predictive

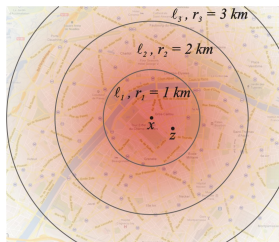
# (In)Distinguishability Metric

What is it that you want to be  
similar to?  
( how much? )



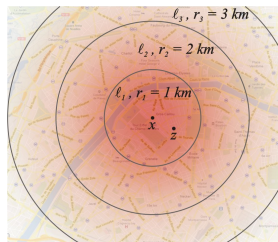
# Euclidean Metric

- Space is privacy
- $\epsilon$  tunes how much



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## Requirement

I want to be indistinguishable from a certain amount of space.

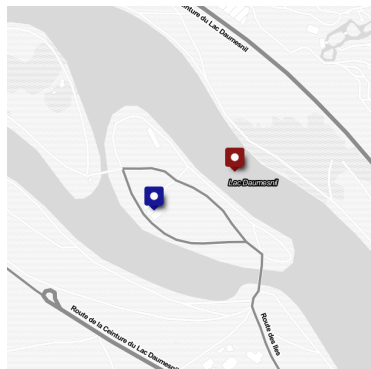
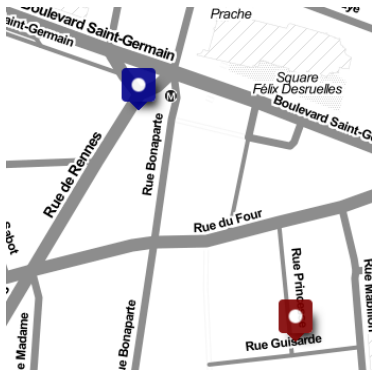
$$req(l)$$

# Problems

- Space is not necessarily privacy...
- Different areas offer different level of privacy

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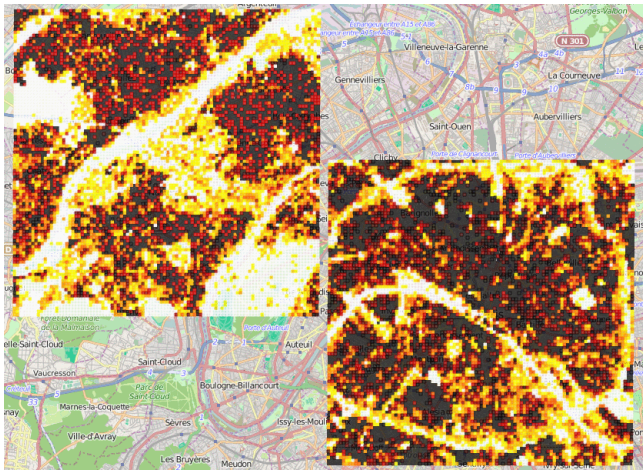
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# OSM enriched Grid

OpenStreetMap: buildings + (POIs x 3)



# Privacy Requirement

## Requirement

I want to be indistinguishable from a certain amount of privacy mass.

We use a quadratic curve (much like for space).

# Building a metric satisfying the requirement

Graph-based algo:

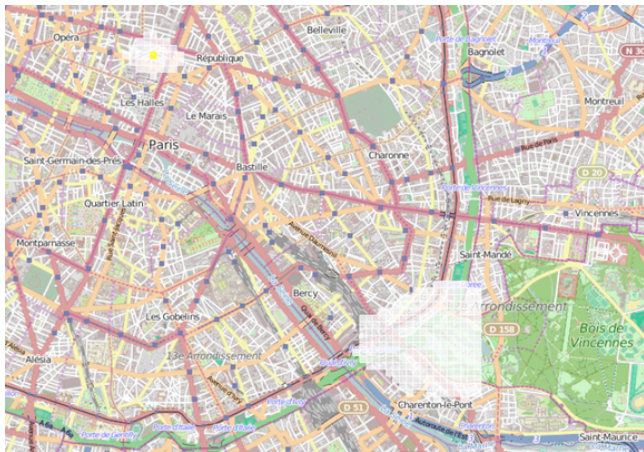
- start with a disconnected graph
- iterate over all nodes
  - ▶ compute `mass`
  - ▶ add an edge with  $l = req^{-1}(\text{mass})$
- we stop at  $l^\top$

# Exponential Mechanism

$$P[z \mid x] \sim e^{-d_{\mathcal{X}}(x,z)}$$

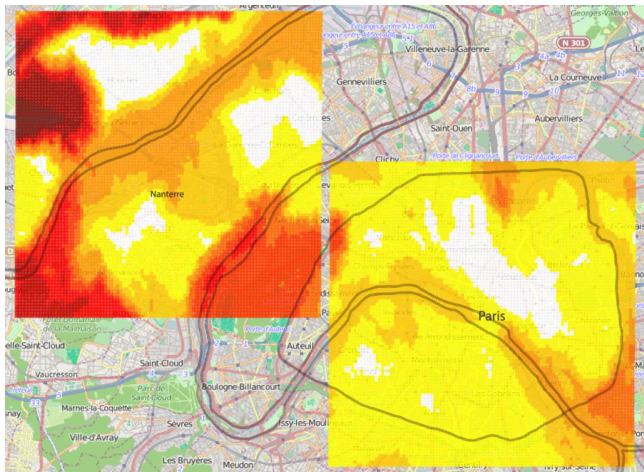
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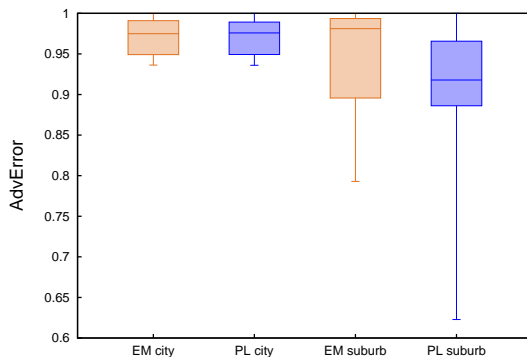
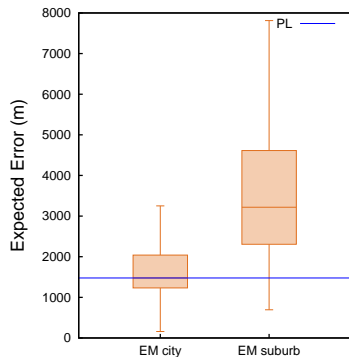
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# Evaluation

- Comparison with geo-indistinguishability
- Fixed Utility as Expected Error
- Compare Privacy as Adversarial Error
- Gowalla and Brightkite datasets

[Shokri, Theodorakopoulos, Boudec, Hubaux. Quantifying location privacy. S&P'11]



# Fences

- linear growth of epsilon
- fences for recurrent places
- achieve “better privacy” consuming less  $\epsilon$

$$d_F(x, x') = \begin{cases} d_X(x, x') & x, x' \notin F \\ 0 & x, x' \in F \\ \infty & o.w. \end{cases}$$





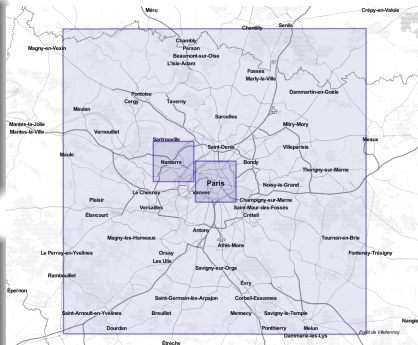
# On the practicality of our method

## Preprocessing

- query osm (highly parallel)
- normalize
- add fences
- build metric (sequential)

## On the phone

- download portion of the map
- compute pdf
- draw



# Tiled Mechanism

Use different  $\epsilon$  in a private way.

